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GENERAL EQUILIBRIUM THEORY WITH MARKET FRICTIONS.

PART I. QUANTITY EQUILIBRIUM WITH RATIONAL EXPECTATIONS.

by

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1. INTRODUCTION

In this paper, I demonstrate the existence of a rational expectations, quantity equilibrium in a general equilibrium model of an economy with market frictions. By an economy with market frictions, I mean an economy in which buyers and sellers have trouble finding each other, it is costly to search for buyers or sellers, and it is costly to wait to buy or sell. By an equilibrium, I mean a stationary probability distribution over the set of possible time paths of states of the economy. This equilibrium reflects rational expectations if all agents in the economy know the stationary distribution of the variables they observe and fully exploit this information. I call the equilibrium a quantity equilibrium because prices are fixed. These prices are not necessarily equilibrium prices, in any sense of the word. They are simply prices which permit the economy to function. Also, there is no uncertainty attached to prices. All agents know all prices and all prices stay constant. There is uncertainty and ignorance, however, about who is ready to buy or sell at any particular time.

My notion of quantity equilibrium is analogous to the rationing equilibria of Drèze [6] and Benassy [1], though rationing in my model results from delays in finding a buyer or a seller. It is not imposed by a rationing scheme, as in the work of Drèze and Benassy.

It may be of some interest to recount my reasons for undertaking this project, for I was influenced by prejudices which I believe are widespread and which I am now tempted to try to abandon. In impressionistic terms, these are the following.

1) An economy may be viewed as a black box with two levers attached, the control and the indicator levers. The control lever is the vector of prices. The indicator lever is the vector of consumer utilities. The box moves the utility lever in response to adjustment of the price lever. One can attach a feedback control mechanism, called the market, to the box and the price lever. If one does so, the utility lever will be moved automatically to a Pareto optimal position.

2) In order that the box and market control model function correctly, it is necessary that the economic actors, who form the mechanism of the box, behave rationally. If one does not assume rational behavior, one introduces spurious inefficiencies into the system which make the market outcome Pareto suboptimal.

3) If there are external effects, incomplete markets, or public goods, the box paradigm fails. In this case, the economist should try to invent corrective controls or adjustments in the mechanism of the box.

4) Arrow-Debreu markets for forward contingent claims are necessary to make the box analogy work in intertemporal models with uncertainty. If these markets do not exist, rational expectations may be used to replace them and to make the market solution Pareto optimal. (This idea is formally correct in special cases and false in others. See Bewley [3]. I doubt that anyone has ever advanced this idea as a general principle.)

When beginning this project, I also intended to criticize the following idea.

5) If one were to incorporate market imperfections and frictions into a formal model, an equilibrium would result which would be Pareto optimal.

This idea may be associated, somewhat unfairly, with Milton Friedman.

For instance, in [8], p. 8, he associates his idea of a natural rate of unemployment with an equilibrium expressible as the solution of the "Walrasian system of general equilibrium equations" for a model with market frictions, imperfect information, etc. As far as I know, he does not claim that the equilibrium would be Pareto optimal.

My plan was to construct a Walrasian model of an economy with market frictions, as Friedman suggests, and then to apply the approach expressed in (1) - (4) to show that (5) makes no sense. In fact, the following two points come to mind.

First of all, there is no Walrasian excess demand equation for a market with frictions. The concept which corresponds to excess demand is the relative size of the queues of unsatisfied buyers and sellers. In a market with frictions, there will always be fluctuating queues of both buyers and sellers, for they must spend time looking for each other. The situation could be, on the average, more favorable to buyers or sellers. Thus, there does exist a qualitative notion of excess demand for markets with frictions. But this notion is imprecise and one cannot easily put one's finger on the size of queues which correspond to equilibrium prices. The notion of equilibrium price must be derived from an adjustment process for prices. It would seem that price adjustment processes do not belong to the domain of Walrasian economics.

The second point is that, as Tobin [11] pointed out, there are many externalities in markets with frictions. More energetic search by one buyer worsens the position of other buyers and improves that of sellers.

There is, in fact, an intriguing problem associated with market frictions. Think of the price in one such market. If the price is high,

sellers are stimulated and buyers are discouraged, so that buyers have an easier time finding the product. This shifts the burden of market frictions to sellers. What should the price be if one takes into account these effects, and how could one recognize the correct price? By exploring this question, I hoped (and still hope) to enrich the point of view expressed by (1)-(3).

In this paper, I construct a general equilibrium model which incorporates market frictions. This is perhaps an accomplishment in itself. But I have been greatly hampered by two problems which indicate that the point of view I started with should probably be changed. The first problem is that rationality of expectations is extravagantly demanding in the context of market frictions. The problems met by economic actors in the presence of frictions are complicated and inherently nonparametric. In order to make a rational decision, an agent should have met many times before any situation which could arise. Only if this is so can he compute the expected benefit from a particular action. This fact leads to many complications, which are discussed in sections 19 and 25. Rational expectations bring on all the awkward and elaborate assumptions made in sections 12, 18, and 19. Rational expectations themselves are hard to define (see section 13).

The second problem is that rational expectations equilibria in models with frictions are not unique and, most important of all, some equilibria may dominate others in the Pareto sense (see sections 23 and 24). This makes it impossible to take the point of view expressed by the black box analogy. In general equilibrium theory, there may also be many equilibria, but these are all Pareto optimal. When one has market frictions, one cannot take it for granted that by appropriate adjustment of prices and so on, one can achieve any sort of optimum. For the optimality of the outcome

depends on expectations, which in turn are mysteriously generated within the system.

These problems lead me to believe that rational expectations and perhaps the black box analogy are a hinderance. Rational expectations prove to be cumbersome and unrealistic. One might even argue that rational expectations are socially bad. The equilibrium of this paper is a state of affairs which exists and is stable only because it has existed and is expected to persist. Rational expectations prevent erratic experimentation which could lead to a switch to a better equilibrium. Be this as it may, the central problem remains that historical conditions may determine the welfare outcome of the system.

I wish to emphasize that I do not regard my work as "explaining" anything in a positivist sense. By constructing a model in which unemployment occurs, I have not discovered why it occurs in reality. My goal is to include an important aspect of reality, market frictions, in the domain of welfare economics.

It should be remarked that the rationing equilibria of Benassy [1] and Drèze [6] also may have multiple equilibria which dominate each other in the Pareto sense. But the rationing equilibria models are essentially different from this one. Rationing equilibria models are general equilibrium models on which one imposes disequilibrium prices and a rationing mechanism. If the prices were in equilibrium, there would be no need for rationing. Hence, from the point of welfare economics, these models are not very interesting. If one wants to improve welfare, one should simply move the prices to an equilibrium position. In the model of this paper, however, rationing is inevitable, no matter what prices may be, and

results from constraints which may be interpreted as physical constraints existing in reality. Thus, the indeterminacy of the equilibria may reflect an unavoidable aspect of reality.

In fact, when one considers that the overall marginal utility of money is not determined, one sees that this indeterminacy may indeed have much to do with current problems. (See section 23 for an explanation of this comment.)

2. SKETCH OF THE MODEL

The construction of a general equilibrium model incorporating market frictions was made possible by what I have previously called the permanent income hypothesis [2]. This amounts to replacing the period-to-period budget constraint by a long-run constraint. Formally, it is assumed that the consumer's marginal utility of money is constant. The consumer satisfies his long-run constraint if his long-run average expenditure equals his long-run average income. This idea makes sense if the economic environment and the consumer's behavior may be described by stationary probability distributions. The permanent income hypothesis makes it unnecessary to keep track of a consumer's money holdings in a model in which his purchases and sales may be widely separated over time.

The model has none of the scope of the usual general equilibrium model. It is almost a minimal model which includes the phenomena I wish to discuss and yet is not a numerical example.

Consumers sell labor, buy goods and earn dividends, as always.

There may be several types of labor; but all consumers can perform all of them. Consumers do all the shopping for goods and jobs. Firms never look for workers or customers. Firms use one type of labor to produce one type of output. They never use goods as inputs. A firm stores its unsold output, which spoils randomly. A firm is ready to sell whatever it has in storage. When it is profitable to hire workers, firms hire the first worker who offers himself. If it is not profitable to keep workers, firms fire them. Firms act so as to maximize their long-run average profit per period.

Time is discrete. In each period, a consumer can ask a particular firm if it will hire him or sell him a unit of its output. The action of asking a firm one of these questions causes disutility. The consumer knows of the existence of each firm, but does not know which have goods in store or job openings. If a consumer is given a job, he continues to work until he quits or is fired. If he buys a good, he buys only one unit. After he asks a firm if it can sell him a unit of output, contact ceases until he again asks the firm a question. Work costs disutility and consumption gives utility. Consumers seek to maximize their long-run average utility flow, where money flows are converted into utility flows at the rate given by the fixed marginal utility of money.

The disutility associated with asking a firm a question may be thought of as associated with the time lost in going to the firm or store. However, for the sake of simplicity no such "housewife time" was included explicitly in the model.

If in any period there is an excess demand for goods or jobs at a firm, a random rationing mechanism comes into play. However, it

should be imagined that this almost never occurs. The time periods are thought of as extremely short, so that it would almost never happen that two consumers would approach a firm at exactly the same moment. There is no queuing. Time should be thought of as nearly continuous. I have used discrete time only to avoid certain technical problems.

Regarding time, it should be imagined that all fluctuations occur rapidly. Financial flows should average out after two years, say. If this attitude is not taken, the permanent income hypothesis does not make sense. Of course, this means that people should change jobs absurdly often. However, it seems worth while to ignore this lack of realism in order to see what happens.

Prices and wages are somewhat arbitrary. Of course, these cannot be such that the economy cannot function at all. There are fines for quitting and for firing and a fee for getting a job. These are novelties which play no role here, but which I hope to exploit in later work.

Fluctuations in consumer and firm behavior occur as a result of fluctuations in production and utility functions and from accumulation or loss of inventories.

An equilibrium is a stationary distribution on the paths of possible states of the economy. Agents' expectations should be rational. The equilibrium is termed long-term if each consumer's marginal utility of money is such that his long-term average expenditure equals his long-term average income.

I do not assume that expectations are completely rational. The description of completely rational expectations becomes too long. I illustrate the difficulties involved by making expectations as to the demand for

and supply of goods and labor fully rational. But the expectations as to quitting and firing are only approximately rational.

Similarly, the conditions guaranteeing that budget constraints are satisfied in equilibrium (section 20) are much too strong. More general conditions were long and distracted from my central points about the indeterminacy of equilibria and the complexity imposed by rational expectations.

A central and very awkward assumption is the ergodicity hypothesis (section 12). It is discussed in section 25.

3. NOTATION

J denotes the set of firms and K the set of consumers, J and K being finite sets. A_i, for $i \in J \cup K$, denotes the set of actions that consumer or firm i can take at any time. C_i, for $i \in J \cup K$, denotes the range of variables pertaining to i which are not controlled by i alone. $A_i \times C_i$ is the set of endogenous states of i. C_i may be thought of as the set of consequences of i's actions.

Throughout this paper, "j" stands for an element of J and "k" for an element of K.

4. THE ENDOGENOUS STATES OF A CONSUMER

The set of actions of consumer k is $A_k = \{0,q\} \cup (J \times \{h\}) \cup (J \times \{b\})$. Here "q" stands for quit, "h" stands for hire and "b" stands for buy. The interpretation of $a_k \in A_k$ is as follows. "$a_k = 0$" means that the consumer

takes no action. "$a_k = q$" means that the consumer quits his job. "$a_k = (j,h)$" means that the consumer asks firm j for a job. "$a_k = (j,b)$" means that the consumer asks firm j if it can sell him one unit of the output it sells.

The set of consequences of consumer k is $C_k = \prod_{h=1}^{5} (J \cup \{0\})$. A point $c_k \in C_k$ is of the form $c_k = (c_{kh}, c_{kf}, c_{kq}, c_{ke}, c_{kb})$, where "f" stands for "fired" and "e" stands for employed. "$c_{kh} = j$" means that the consumer is hired by j during the period. "$c_{kh} = 0$" means that he is not hired. "$c_{kf} = j$" means that the consumer is fired by firm j. "$c_{kf} = 0$" means that he is not fired. "$c_{kq} = j$" means that the consumer quits firm j during the period. "$c_{kq} = 0$" means that he does not quit. "$c_{ke} = j$" means that the consumer is employed by firm j at the end of the period. "$c_{ke} = 0$" means that he is unemployed at the end of the period. "$c_{kb} = j$" means that the consumer buys one unit of output from firm j during the period. "$c_{kb} = 0$" means that he does not buy.

5. THE ENDOGENOUS STATES OF A FIRM

The set of actions of firm j is $A_j = \{-1, 0, 1\}$. If $a_j \in A_j$, then "$a_j = -1$" means that the firm fires one worker. "$a_j = 0$" means that it neither fires nor hires, and "$a_j = 1$" means that it tries to hire one worker.

The set of consequences for firm j is $C_j = \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1,\ldots,B\} \times \{0,1,\ldots,B\} \times \{0,1\} \times \{0,1\} \times \{0,1,\ldots,B\} \times \{0,1,\ldots,B\}$, where B is a large positive integer which exceeds the number of consumers, $|K|$. A point $c_j \in C_j$ is of the form $c_j = (c^o_{jh}, c_{jh}, c_{jf}, c_{jq}, c_{je}, c^o_{jb}, c_{jb}, c_{jp}, c_{js})$, where "h" stands for hired, "f" for fired, "q" for quit, "e" for employed, "b" for bought, "p" for production and "s" stands for stocks of goods. The

interpretation is as follows. $c^o_{jh} = 1$ if at least one consumer offered to work for the firm during the period. Otherwise $c^o_{jh} = 0$. c_{jh} is the number of workers hired during the period. c_{jf} is the number of workers fired during the period. c_{jq} is the number of workers who quit during the period. c_{je} is the number of workers employed by the firm at the end of the period. $c^o_{jb} = 1$ if at least one consumer tried to buy during the period. Otherwise, $c^o_{jb} = 0$. c_{jb} is the quantity of output sold during the period. c_{jp} is the quantity produced during the period. c_{js} is the stock of goods held at the end of the period.

6. EXOGENOUS UNCERTAINTY

The behavior of consumers and firms is influenced by random variation. s_{in}, for $i \in J \cup K$, denotes the random variable observed by agent i during period n. S_{0i} denotes the range of s_{in}. That is, $s_{in} \in S_{0i}$. s_{rn} is a random variable governing rationing at time n. It is assumed that

(6.1) $\{s_{rn}\}_{n=-\infty}^{\infty}$ and $\{s_{in}\}_{n=-\infty}^{\infty}$, for $i \in J \cup K$, are Markov processes.

s_{rn} and s_{in} are generated as follows. There is an underlying stochastic process $\{s_n\}_{n=-\infty}^{\infty}$, where s_n belongs to a measurable space S_0. $s_r : S_0 \to [0,1]$ and $s_i : S_0 \to S_{0i}$ are measurable functions. $s_{rn} = s_r(s_n)$, and $s_{in} = s_i(s_n)$, for all i.

$S = \prod_{n=-\infty}^{\infty} S_0$ denotes the sample space of the process $\{s_n\}$. S is given the product measurable structure. The probability law governing

the process $\{s_n\}$ is a Borel measure ν in S. It is assumed that

ν is stationary.

That is, $\nu(\tau_S E) = \nu(E)$, for all measurable subsets E of S, where $\tau_S = S \to S$ is the shift operator defined by $\tau_S(s)_n = s_{n+1}$.

7. ECONOMIC STEADY STATES

The state of the economy at one moment is described by a point in $S_0 \times \prod_{i \in J \cup K} (A_i \times C_i)$. The set of all paths of states is the space $M = \prod_{n=-\infty}^{\infty} [S_0 \times (\prod_{i \in J \cup K} (A_i \times C_i))]$. A point in M is denoted by $(s, (a_i, c_i)_{i \in J \cup K})$, with n^{th} component $(s_n, (a_{in}, c_{in})_{i \in J \cup K}) \in S_0 \times \prod_{i \in J \cup K} (A_i \times S_i)$. $\prod_{i \in J \cup K} (A_i \times C_i)$ is a finite set, so that $S_0 \times \prod_{i \in J \cup K} (A_i \times C_i)$ is a finite disjoint union of measurable spaces and hence is also measurable. M is given the product measurable structure.

A steady state for the economy is a stationary stochastic process in M which is consistent with the distribution, ν, of the exogenous process $\{s_n\}_{n=-\infty}^{\infty}$. That is, a steady state is a Borel probability μ in M such that

(7.1) $\mu\{(s,(a_i, c_i)_{i \in J \cup K}) \in M \mid s \in B\} = \nu(B)$, for all measurable subsets B of S, and

(7.2) $\mu(\tau_M E) = \mu(E)$, for all measurable subsets E of M,

where $\tau_M : M \to M$ is the shift operator defined by $\tau_M(s,(a_i, c_i)_{i \in J \cup K})_n = (s_{n+1}, (a_{i,n+1}, c_{i,n+1})_{i \in J \cup K})$.

8. THE INTERACTION OF CONSUMERS AND FIRMS

In this section, I describe how the consequence variables evolve in response to the actions of consumers and firms. The actions and consequences in period n are denoted by a_{in}, c_{ihn}, c_{ifn}, and so on, for $i \in J \cup K$. The variables c_{jpn} and c_{jsn}, for $j \in J$, are determined by the technical conditions of firms, described in section 10. Here, I describe the determination of the other consequence variables. Let $k \in K$.

$\underline{c_{khn}}$. Consumer k is hired by firm j (i.e., $c_{khn} = j$), only if the following conditions are satisfied. 1) $a_{kn} = (j,h)$ (i.e., k asks for the job), 2) $c_{ke,n-1} \neq j$ (i.e., k is not already employed by j) and 3) $a_{jn} = 1$ (i.e., firm j is hiring). If these conditions are satisfied and if consumer k is the only worker satisfying these conditions, he gets the job (i.e., $c_{khn} = j$). If several workers satisfy these conditions, one of them is given the job at random. All the job applicants are given the job with equal probability. The random choice of employee is made according to s_{rn}.

$\underline{c_{kfn}}$. Firm j fires an employee during period n (i.e., $c_{kfn} = j$) only if $a_{jn} = -1$. If the firm has only one employee (i.e., if $c_{j,n-1} = 1$), that employee is dismissed. If the firm employs several workers, one is chosen at random and dismissed. The choice is made according to s_{rn} and all employees are equally likely to be fired.

$\underline{c_{kqn}}$. Consumer k quits his job at firm j (i.e., $c_{kqn} = j$) if and only if $c_{ke,n-1} = j$ and $a_{kn} = q$.

$\underline{c_{ken}}$. If consumer k is hired by firm j, he is employed by j, even if he previously worked for another firm. That is, $c_{ken} = j$, if $c_{khn} = j$.

If k is not hired and if he is fired or quits, then he becomes unemployed. In symbols, $c_{ken} = 0$ if $c_{khn} \neq j$, for some $j \in J$, and if $c_{kfn} = j$ or $c_{kqn} = j$, for some $j \in J$. If k does not quit and is neither hired nor fired, then his employment status does not change. That is, if $c_{khn} = c_{kfn} = c_{kqn} = 0$, then $c_{ken} = c_{ke,n-1}$.

$\underline{c_{kbn}}$. Consumer k buys one unit of output from firm j (i.e., $c_{kbn} = j$) only if he offers to buy and firm j has output in stock. I.e., $c_{kbn} = j$ only if $a_{kn} = (j,b)$ and $c_{js,n-1} > 0$. If consumer k is the only consumer satisfying these conditions, he buys. If several consumers do so, then one is chosen at random according to s_{rn} and is allowed to buy. All are chosen with equal probability.

Now let $j \in J$.

$\underline{c_{jhn}, c_{jfn}, c_{jqn}, \text{ and } c_{jbn}}$. For y = h, f, q, or b, $c_{jyn} = 1$ if and only if $c_{kyn} = j$ for some $k \in K$. Otherwise, $c_{jyn} = 0$.

$\underline{c_{jen}}$. $c_{jen} = |\{k \in K : c_{ken} = j\}|$, where "$|\cdot|$" stands for cardinality. Notice that c_{jen} cannot exceed its assumed upper bound, B, since $B > |K|$.

$\underline{c^o_{jhn} \text{ and } c^o_{jbn}}$. $c^o_{jyn} = 1$ if and only if $a_{kn} = (j,y)$, for some $k \in K$, where y = h or b. Otherwise, $c^o_{jyn} = 0$.

9. WAGES AND PRICES

Money is a commodity distinct from labor and produced goods. The stocks of money held by various agents never appear in the model. All accounting is in terms of flows of money.

Workers pay firms a fixed price for their output and receive a wage, both in terms of money. p_j is the price paid by consumers for one unit of the output of firm j. w_j is the wage paid by firm j for one period of labor.

Associated with each job, there is not only a wage but also fees for hiring, firing and quitting. These fees have to do with the fact that a job has a value, for both the firm and the worker, which is distinct from the value of the wages paid and the work performed. The firm and the employee are in contact and need each other. If this contact were broken, it might take some time and effort for new contacts to be established. The fee paid by a worker to firm j when he is hired by j is w_{hj}. The fee paid by a worker to firm j if he quits his job there is w_{qj}. Finally, firm j pays a worker w_{fj} if he fires him. w_{qj} and w_{fj} are non-negative. w_{hj} may be of either sign.

All prices and fees are known to all consumers and firms.

10. FIRMS

The amount produced by firm j during period n, c_{jpn}, is a random function of the labor input of the previous period.

$$(10.1) \qquad c_{jpn} = g_j(s_{jn}, c_{je,n-1}), \text{ for all } j \in J.$$

This means that production is essentially instantaneous. It is possible to make output depend on earlier inputs of labor, but the lags greatly complicate the notation.

It is assumed that output spoils randomly, this spoilage occurring after sales have been made. For the moment, let ℓ_{jn} be the spoilage of firm j during period n. Then,

$$(10.2) \qquad \ell_{jn} = L_j(s_{jn}, c_{js,n-1} - c_{jbn}), \text{ where}$$

$$0 \leqq \ell_{jn} \leqq c_{js,n-1} - c_{jbn}.$$

The firm can hold no more than B units of output. Hence, the stocks of firm j at the end of period n are

$$(10.3) \qquad c_{jsn} = \min(B, c_{js,n-1} + c_{jpn} - c_{jbn} - \ell_{jn}).$$

The profit of firm j during period n is

$$(10.4) \qquad r_{jn} = r_j(c_{jn}) = p_j c_{jbn} - w_j c_{jen} + w_{qj} c_{jqn} + w_{hj} c_{jhn} - w_{fj} c_{jfn}.$$

The first term represents revenues from sales, the second represents wage payments, the third fees collected from quitters, the fourth hiring fees, and the fifth represents fines paid for firing.

The objective of the firm is to maximize its long-run average profit per period. In section 14, it is proved that the firm has a stationary optimal policy.

11. CONSUMERS

It is assumed that the consumer receives a random flow of utility in each period, which depends on his purchase of goods during the previous period. He also suffers a random disutility from working and another disutility from hunting for jobs or goods. Formally, the net flow to consumer k in period n is

$$(11.1) \qquad u_{kn} = U_k(s_{kn}, a_{kn}, c_{kn})$$

$$= U_{kb}(s_{kn}, c_{kbn}) - U_{ke}(s_{kn}, c_{ken}) - U_{kA}(s_{kn}, a_{kn}).$$

U_{kb} is the flow of utility from consumption. U_{ke} is the flow of disutility from working. U_{kA} is the flow of disutility from acting.

It is assumed that

$$U_{kb}(s,0) = U_{ke}(s,0) = U_{kA}(s,0) = 0, \quad \text{for all } s \in S_{0k}$$

and for all k. All these functions are non-negative.

The net revenue (excluding dividends) of consumer k during period n is

(11.2) $$r_{kn} = \hat{c}_{khn} + \hat{c}_{kfn} + \hat{c}_{kqn} + \hat{c}_{ken} + \hat{c}_{kbn}\,,$$

where the $\hat{c}_k$ variables are defined as follows.

$$\hat{c}_{khn} = \begin{cases} -w_{hj}\,, & \text{if } c_{khn} = j \text{ for } \quad j \in J, \\ 0\,, & \text{otherwise.} \end{cases}$$

$$\hat{c}_{kfn} = \begin{cases} w_{fj}\,, & \text{if } c_{kfn} = j \text{ for } \quad j \in J, \\ 0\,, & \text{otherwise.} \end{cases}$$

$$\hat{c}_{kqn} = \begin{cases} -w_{qj}\,, & \text{if } c_{kqn} = j \text{ for } \quad j \in J, \\ 0\,, & \text{otherwise.} \end{cases}$$

$$\hat{c}_{ken} = \begin{cases} w_{j}\,, & \text{if } c_{ken} = j \text{ for } \quad j \in J, \\ 0\,, & \text{otherwise.} \end{cases}$$

$$\hat{c}_{kbn} = \begin{cases} -p_{j}\,, & \text{if } c_{kbn} = j \text{ for } \quad j \in J, \\ 0\,, & \text{otherwise.} \end{cases}$$

Using the permanent income hypothesis, I assume that consumer k can convert money freely to utility at a constant rate of λ_k units of utility per unit of money. The net utility flow to consumer k in period n is

(11.3) $$u_{kn} + \lambda_k r_{kn}\,.$$

The objective of the consumer is to maximize the expected long-run average flow of $u_{kn} + \lambda_k r_{kn}$. The proof that a consumer has an optimal policy appears in section 14.

12. THE ERGODICITY HYPOTHESIS

For several reasons, I make a bizarre assumption which guarantees that the equilibrium generated by the economy will be ergodic. Two of these reasons are explained in section 25. The assumption also makes it possible to prove that firms and consumers have optimal policies (see section 14).

The assumption is the following. In any period, the whole economy stops with a certain small probability, starting up again a short time later. The important point is that events occurring before the interruption do not affect what happens afterward.

In detail, the assumption is the following. S_0, the set of exogenously determined states, contains two sorts of elements, "ordinary" states and "carnival" states. There are three carnival states, s_0^*, s_1^*, and s_2^*. The evolution of the exogenous process $\{s_n\}$ obeys the following rules. If $s_n = s_t^*$, for t = 0 or 1, then $s_{n+1} = s_{t+1}^*$. If $s_n = s_2^*$, then $s_{n+1} \in S_0 \setminus \{s_0^*, s_1^*, s_2^*\}$. If s_n is an ordinary state, then $s_{n+1} = s_0^*$ with probability $\delta > 0$ and s_{n+1} never equals s_1^* or s_2^*.

It is assumed that all agents observe the carnival states. More formally, in the language of section 6,

$$(12.1) \qquad s_t^* \in S_{0i} \text{ and } s_t^* = s_i(s_t^*) \text{ for all } i \in J \cup K, \quad t = 0, 1, 2.$$

I also make the following assumptions which guarantee that an economy stops during a carnival.

(12.2) If s_n is an ordinary state, then, for all $k \in K$, $U_k(s_n, a, c) < \infty$ for all $(a, c) \in A_k \times C_k$.

However, if s_n is a carnival state, the following are true.

(12.3) For all $k \in K$, $U_{ke}(s_{kn}, c) = \infty$ if $c \neq 0$. That is, work is infinitely costly.

(12.4) For all $k \in K$, $U_{kA}(s_{kn}, a) < \infty$, for all $a \in A_k$. That is, action is not infinitely costly.

(12.5) For all $k \in K$, $U_{kb}(s_{kn}, c) = 0$, for all c. That is, consumption is useless.

(12.6) For all $j \in J$, $g_j(s_{jn}, c) = 0$ and $L_j(s_{jn}, c) = c$, for all c. That is, there is no production and all stocks of goods spoil.

It follows from (12.2)-(12.6) that if $s_n = s_0^*$, then it is rational for all workers to quit their jobs immediately and take no action during periods n+1 and n+2. Also, it does not make sense for a firm to offer work. In fact, $(a_{in}, c_{in})_{i \in J \cup K} = 0$ is a rational outcome whenever $s_n = s_1^*$ or s_2^*.

13. EXPECTATIONS

I here define the expectations of firms and consumers. As stated before, the expectations as to quitting and firing are not fully rational. Expectations as to demand and supply are treated as fully rational, given that observations of quitting and firing are ignored in the calculation of those expectations.

I make the following general assumptions.

(13.1) Agents observe only their own state.

(13.2) Agents assume that there is no correlation between events before and after a carnival.

(13.3) Each agent, i, assumes that the evolution of events external to him is statistically independent of his own exogenous random variable, s_{in}, and of his own actions. (An exception occurs if s_n is a carnival state. In this case, all agents know that everything will stop.)

Assumption 13.2 is self-fulfilling, since expectations provide the only link that could exist between events before and after a carnival (see 6.1). Assumption 13.3 resembles an assumption of perfect competitiveness.

In what follows, it will be assumed that the economic steady state distribution, μ, is given (see section 7).

Rational Expectations of Firms

I here define the firm's anticipation of demand for his output and supply of labor. It is assumed that the only relevant information is the following: the time elapsed since the last carnival, past observations of the variables c^o_{jhn} and c^o_{jbn} and knowledge of their distribution, as determined by μ. This assumption is in accord with (13.1) - (13.3).

The observation of firm j at time n, (c^o_{jhn}, c^o_{jbn}), is denoted by y_{jn}. The observational history of j at time n, H^o_{jn}, is the history of y_{jn} since the last time the exogenous state was s^*_1. That is, $H^o_{jn} = (y_{j,n-t}, \ldots, y_{j,n-1})$, where t is the smallest positive integer such that $s_{n-t} = s^*_1$.

Suppose that at the beginning of period n the observational history of firm j is $\bar{H}$. Then, firm j assumes that during period n, y_{jn} will equal a possible value y with the probability given by μ to the event $[y_{jn} = y]$ conditional on the event $[H^o_{jn} = \bar{H}]$. In symbols,

$$\text{Prob}\,[y_{jn} = y] = \mu\,[y_{jn} = y \mid H^o_{jn} = \bar{H}].$$

In order that this conditional probability make sense, it must be that the conditioning event have positive probability. Let $\mathscr{H}^o_j = \{(x_1, \ldots, x_{2N}) \mid x_n = 0 \text{ or } 1 \text{ for all } n \text{ and } x_1 = x_2 = x_3 = x_4 = x_6 = 0;\ N \geqq 1\}$ be the set of observational histories of firm j. (The zeros have to do with the fact that nothing happens during a carnival.) Then, I must suppose that

(13.4) (Complete expectations) $\mu\,[H^o_{jn} = H] > 0$, for all $H \in \mathscr{H}^o_j$.

The necessity of this hypothesis is one of the difficulties caused by non-parametric rational expectations. (13.4) is justified in section 19.

<u>Rational Expectations of Consumers</u>

The consumer's anticipations of demand and supply are more complicated than the firm's, for the information he has depends on his own actions. He learns about market conditions only if he offers to buy or sell.

The <u>observations</u> of the consumer are the success or failure of his attempts to buy or sell. y_{kn} will stand for the observation of k at time n. It is determined as follows. $y_{kn} = 0$, if $a_{kn} = 0$ or q (i.e., if k does not try to buy a good or to get a job). $y_{kn} = (a_{kn}, c_{kbn})$, if $a_{kn} = (j,b)$. And $y_{kn} = (a_{kn}, c_{khn})$, if $a_{kn} = (j,h)$.

The <u>observational history</u> of k at time n is $H^o_{kn} = (y_{k,n-t}, \ldots, y_{k,n-1})$, where t is the smallest positive integer such that $s^*_{n-t} = s^*_1$.

Consumers believe that they observe whether or not firms have goods and jobs available. That is, I ignore the fact that a consumer may fail to get a job or good because someone else has applied for it at the same time. (Recall that time periods are short.) Let $z_{jn} = (z_{jhn}, z_{jbn})$ be a variable reflecting conditions at firm j, where

$$z_{jhn} = \begin{cases} 1, \text{ if } a_{jn} = 1 \text{ (i.e., j is hiring)} \\ 0, \text{ if } a_{jn} = 0 \text{ or } -1. \end{cases}$$

$$z_{jbn} = \begin{cases} 1, \text{ if } c_{js,n-1} > 0 \text{ (i.e., j can sell)} \\ 0, \text{ if } c_{js,n-1} = 0. \end{cases}$$

Let $z_n = (z_{jn})_{j \in J}$ and let $Z_n = (z_{n-t}, \ldots, z_{n-1})$, where t is the smallest positive integer such that $s_{n-t} = s_1^*$. Let Ω be the set of possible values of Z_n. That is, $\Omega = \{((x_{jh1}, x_{jb1})_{j \in J}, \ldots, (x_{jhN}, x_{jbN})_{j \in J} \mid x_{jhn}$ and x_{jbn} equal 0 or 1 for all j and n and $x_{jh1} = x_{jb1} = x_{jh2} = x_{jb2} = x_{jb3} = 0 ; N \geqq 1\}$.

Consumer k interprets y_{kn} as an observation of z_n in an obvious way. For instance, if $y_{kn} = ((j,b),j)$, then k knows that $z_{jbn} = 1$. Thus, the statement "$H^o_{kn} = \overline{H}$" is a statement about Z_n and so is an event defined on Ω. This event is denoted by $[H^o_{kn} = \overline{H}]$. Similarly, the event $[y_{kn} = \overline{y}]$ is a statement about z_n.

The consumer is assumed to know the distribution of Z_n (as a result of previous experimentation, perhaps). His rational expectation given his observational history is $\overline{H}$ and is given by the following formula.

$$\text{Prob}[y_{kn} = y] = \mu[[y_{kn} = y] \mid [H^o_{kn} = \overline{H}]].$$

In order that this conditional probability make sense, I must assume the following.

(13.5) (Complete expectations) $\mu[Z_n = \bar{Z}] > 0$ for all $\bar{Z} \in \Omega$.

This assumption is justified in section 19.

Expectations as to Quitting or Firing

Each consumer believes that the event that he is fired is independent of all observations (save the fact that he must be employed in order to be fired). Similarly, firms believe that the event that an individual worker quits is independent of all observations and of the event that any other worker quits. An exception must be made if it is a period of carnival. Then, the firm knows that all workers will quit.

The probability of quitting anticipated by firm j is $E_\mu c_{jqn}/E_\mu c_{jen}$, where E_μ stands for expectation with respect to μ. Each consumer anticipates that the probability of being fired by firm j if he works there is $E_\mu c_{jfn}/E_\mu c_{jen}$. For these definitions to make sense, it must be that

(13.6) $E_\mu c_{jen} > 0$, for all j.

14. EXISTENCE OF OPTIMAL POLICIES

The ergodicity hypothesis and the formulation of rational expectations imply that each agent's optimization problem has the form of a Markov decision process. This process is described as follows. Let $i \in J \cup K$.

The set of actions of agent i is simply A_i.

In order to define the state space of agent i, let H_{in} be the history at time n of the endogenous states of agent i since the first period of the last carnival. That is, $H_{in} = (a_{i,n-t}, c_{i,n-t}, \ldots, a_{i,n-1}, c_{i,n-1})$, where t is the smallest positive integer such that $s_{n-t} = s_1^*$. Let $\mathscr{H}_i$ be the set of all

possible histories of i. Let S_{0i}, the exogenous states of i, be as defined in section 6. The state space of the process is $S_{0i} \times \mathscr{H}_i$. The state of agent i at time n is (s_{in}, H_{in}).

The <u>transition probabilities</u> of the process are defined by agent i's expectations and by the transition probabilities for the exogenous Markov process $\{s_{in}\}$. Let $(s_{in}, H_{in}) \in S_{0i} \times \mathscr{H}_i$ be the state at time n. $\pi(s_{in}, ds)$ denotes the probability distribution of $s_{i,n+1}$ given s_{in}. $H_{i,n+1}$ depends on H_{in}, the actions of the agent, and the random outcome of the action. Since the agent's observational history, H^o_{in}, is included in H_{in}, the agent's rational expectations may be derived from H_{in}. $q_i(a,H;H')$ denotes the

probability of the succeeding history, H', given that the action is $a \in A_i$, the current history is H, and the agent is not in a carnival period. q_i is calculated according to the expectations defined in section 13. q_i depends on the assumed steady state distribution, μ, of course.

An exception occurs in the definition of the transition probabilities if $s_{in} = s^*_1$. Then, all components of H_{in} corresponding to periods preceding period n are removed. Recall that in this case, $s_{i,n+1} = s^*_2$. Also, since nothing happens during a carnival, all the components of $H_{i,n+1}$ are zero. Call this one period history of zero, H^*. (s^*_2, H^*) is a recurrent state in the process.

The <u>reward</u> of agent i is given by formulas 10.4 and 11.4. The reward in period n depends on s_{in}, a_{in}, and c_{in}, that is, on s_{in} and $H_{i,n+1}$. This reward is denoted by $\rho_i(s_{in}, H_{i,n+1})$.

I use dynamic programming to prove that an optimal policy exists. Let $V_i(s,H)$ be the value of operating the decision process until the state

(s_2^*, H^*) is reached, given that the current state is (s, H). Clearly, $V_i(s_2^*, H^*) = 0$. $V_i(s_0^*, H)$ is computable for all H, since (s_0^*, H) goes in two steps to (s_2^*, H^*). Let $\hat{S}_{0i} = S_{0i} \setminus \{s_0^*, s_1^*, s_2^*\}$ be the set of ordinary states. On $\hat{S}_{0i}$, V_i obeys the following equation, since the process always moves to s_0^* with probability $\delta > 0$.

$$(15.1) \qquad V_i(s,H) = \max_{a \in A_i} \sum_{H' \in \mathscr{H}_i} q_i(a,H;H') \Big[\rho_i(s,H') + \delta V_i(s_0^*,H') + (1-\delta) \int_{\hat{S}_{0i}} V_i(s',H')(1-\delta)^{-1} \pi_i(s,ds') \Big],$$

for $s \in \hat{S}_{0i}$.

Let L be the space of all bounded real-valued measurable functions on $\hat{S}_{0i} \times \mathscr{H}_i$ and give L the supremum norm. L is then a Banach space [7], p. 258. If $V \in L$, let $T(V) \in L$ be defined by the right-hand side of (15.1), with $V(s',H')$ substituted for $V_i(s',H')$. (The expression $V_i(s_0^*,H')$ must be left unchanged, of course.) Since A_i is finite, $T(V)$ is well-defined and measurable. It is certainly bounded. $T : L \rightarrow L$ is clearly contracting, for

$$\| T(V^1 - V^2) \| \leqq (1-\delta) \| V^1 - V^2 \|,$$

where "$\| \cdot \|$" denotes the supremum norm. By the contracting map fixed point theorem [9], p. 33, T has a unique fixed point, V_i. This function clearly satisfies (15.1). This proves that V_i is well-defined. It follows immediately that agent i has an optimal policy.

An <u>optimal</u> <u>policy</u> for agent i may be represented as a measurable function $a_i : S_{0i} \times \mathscr{H}_i \rightarrow A_i$. The choice of i at time n is $a_i(s_{in}, H_{in})$.

It is important to recall that the optimal policy a_i is optimal given that μ is the steady state distribution of the economy.

15. STEADY STATES GENERATED BY POLICIES

I now fix policies $a_i : S_{0i} \times \mathscr{H}_i \to A_i$, for $i \in J \cup K$ and show that there results a unique steady state distribution, μ, for the economy.

The <u>history</u> of the economy at time n is $H_n = (H_{in})_{i \in J \cup K}$. Let $\mathscr{H}$ be the set of all feasible histories for the economy. The policies a_i and the evolution of $\{s_{in}\}$ define a Markov process $\{s_n, H_n\}$ on $S_0 \times \mathscr{H}$. The only complication arises when $s_n = s_1^*$. Then, as in the previous section, all the components of H_n corresponding to periods preceding period n are dropped when composing H_{n+1}. Also, in this case all the components of H_{n+1} are zero. Call this state H^* again.

(s_2^*, H^*) is a recurrent state of the Markov process. In fact, if s_n is not a carnival state, then with probability $\delta > 0$, (s_2^*, H^*) is reached from (s,H) in three periods. Hence, the Markov process $\{s_n, H_n\}$ has a unique stationary distribution. This distribution induces, in an obvious manner, a unique stationary distribution in

$$M = \prod_{n=-\infty}^{\infty} \left(S_0 \times \prod_{i \in J \cup K} (A_i \times C_i)\right).$$

Call this distribution $\mu(a)$, where $a = (a_i)_{i \in J \cup K}$. It is clear that $\mu(a)$ is an economic steady state satisfying (7.1) and (7.2).

Since the state (s^*, H^*) is recursive for the steady state distribution on $S_0 \times \mathscr{H}$, $\mu(a)$ is ergodic. That is, all $\mu(a)$-invariant sets of M have probability zero or one. A set $E \subset M$ is $\mu(a)$-invariant if $\mu(a)(E \setminus \tau_M E) = 0$,

where $\tau_M = M \to M$ is the shift operator. (See Doob [5], pp. 457-460, for a discussion of ergodicity. He calls it, metric transitivity.)

16. THE BUDGET CONSTRAINT

Every steady state distribution for the economy, μ, determines an expected profit for each firm, $r_j(\mu) = E_\mu(r_{jn})$, where r_{jn} is as in (10.4). Since μ is stationary, $E_\mu(r_{jn})$ does not depend on n. If μ is ergodic, then by the strong law of large numbers, $E_\mu(r_{jn})$ is also the long-run average profit of the firm. (See Doob [5], p. 465, for the version of the strong law of large numbers referred to.) Since the equilibrium μ will be ergodic, $r_j(\mu)$ is an appropriate definition of the long-run profit of the firm.

It is assumed that in each period consumer k receives a share θ_{kj} of $r_j(\mu)$, where $0 \leqq \theta_{kj} \leqq 1$ and $\sum_{k\in K} \theta_{kj} = 1$, for all j.

The long-run net revenue of consumer k, excluding dividends, is $r_k(\mu) = E_\mu(r_{kn})$, (see 11.2). Hence, the long-run net revenue of consumer k is

$$\beta_k(\mu) = r_k(\mu) + \sum_{j\in J} \theta_{kj} r_j(\mu). \tag{16.1}$$

The long-run budget constraint of the consumer is $\beta_k(\mu) \geqq 0$.

17. THE DEFINITION OF EQUILIBRIUM

Throughout the following, $\lambda = (\lambda_k)_{k\in K}$ will denote the vector of marginal utilities. It is assumed that

(17.1) $\lambda_k \geqq 0$, for all k, and $\sum_{k \in K} \lambda_k = 1$.

A short-term equilibrium for the economy, given a vector of marginal utilities λ, is a steady state distribution, μ, on M satisfying (17.2) - (17.4) below.

(17.2) $\mu = \mu(a)$, where $a = (a_i)_{i \in J \cup K}$ is a list of policies.

(17.3) For all $i \in J \cup K$, a_i is an optimal policy for i, given expectations based on μ.

(17.4) $E_\mu(\hat{c}_{kbn}) > 0$, for some $k \in K$,

where $\hat{c}_{kbn}$ is the expenditure of consumer k on consumption during period n (see section 11). Condition 17.4 guarantees that the equilibrium is not trivial. That is, it guarantees that some economic activity takes place.

A long-term equilibrium is a vector of marginal utilities λ and a distribution μ, such that μ is a short-term equilibrium given λ and

(17.5) $b_k(\mu) = 0$, for all $k \in K$.

A short-term equilibrium is, perhaps, the appropriate concept, since the equilibrium of this paper is meant to reflect the nature of an economy over a short period of time. It must be assumed that the consumers' action functions change slowly over time in response to changes in the exogenous environment or their marginal utilities of money. If condition 17.5 were not satisfied, consumers would eventually observe this fact and adjust their marginal utilities of money. For instance, one can imagine that if $b_k(\mu)$ were positive, consumer k would slowly reduce

λ_k. (See Bewley [4] for a discussion of the aggregate stability of this adjustment process.)

18. UNIQUENESS OF OPTIMAL POLICIES

It is important that the optimal choices of every agent be almost surely unique. If they are not unique, the response of agents to changes in the steady state distribution, μ, can be discontinuous and quantity equilibria may not exist. For instance, if the probability of finding output at firms 1 and 2 were the same, a consumer might choose either with equal probability. But if the probability of finding the product at one firm were slightly increased, he would shop there first.

I make a series of forced and artificial assumptions in order to avoid this difficulty. This problem is one of the major difficulties imposed by rational expectations.

First, I deal with the choices of agents during carnival periods. I assume the following.

(18.1) For all $i \in J \cup K$, if $s_{in} = s_1^*$ or s_2^*, or if $s_{in} = s_0^*$ and $i \in J$, then $a_i(s_{in}, H_{in}) = 0$. If $s_{in} = s_0^*$ and $i \in K$, then $a_i(s_{in}, H_{in}) = q$ if i is working and 0 if he is not.

By (12.1)-(12.6), the behavior described by (18.1) is optimal.

The following strong assumptions make consumer choices almost surely unique in non-carnival states. They assert that the immediate cost of making a decision, $U_{kA}(s_{kn}, a_{kn})$, is independent of future rewards and has a distribution which is so dispersed that it breaks ties almost surely.

(18.2) For every $k \in K$ and $a \in A_k$, if s_n is not a carnival state, then $U_{kA}(s_{kn}, a)$ and s_{km} are mutually independent for all $m > n$.

(18.3) For all $k \in K$ and for all real numbers r,
$$\text{Prob}\,[U_{kA}(s_{kn}, a) - U_{kA}(s_{kn}, a') = r \,|\, s_{kn} \notin \{s_0^*, s_1^*, s_2^*\}] = 0,$$
for all distinct points a and a' in A_k.

In order to make firms' choices unique, I assume that the expected productivity of workers next period fluctuates sufficiently in a way which is independent of what happens next period. More precisely, I make the following assumptions for all $j \in J$ and for all non-carnival periods.

(18.4) $s_{jn} = (s_{j1n}, s_{j2n})$, where s_{j1n} is independent of s_{jm}, for all $m > n + 1$.

(18.5) s_{j1n} is independent of $L_j(s_{jn}, \cdot)$ and $L_j(s_{j,n+1}, \cdot)$. (See 10.2 for a definition of L_j.)

(18.6) $E(g_j(s_{j,n+1}, c) \,|\, s_{jn}) = cG_{j1}(s_{jn}) + G_{j2}(s_{jn})$, for all c. (See 10.1 for a definition of g_j.)

(18.7) For all real numbers r,
$$\text{Prob}\,[G_{j1n}(s_{jn}) = r \,|\, s_{j2n}] = 0.$$

I add the following restriction on the firm's decisions.

(18.8) For all j, $a_{jn} \geqq 0$ if $c_{je,n-1} = 0$.

That is, a firm never decides to fire somebody when he has no employees. Since this action would be pointless, (18.8) accords with rational behavior.

It is easy to verify that (18.4) - (18.8) guarantee almost sure uniqueness, provided that output always has value. This will be the case. In fact, the complete expectations hypothesis (13.4) guarantees that in any non-carnival period, any firm can always expect to have some demand in the following period.

19. THE COMPLETE EXPECTATION HYPOTHESES

The complete expectation hypotheses (13.4) - (13.6) have to do with the most awkward point in the model. If rational expectations are to make sense, no agent must ever be faced with a situation he does not meet regularly. In a steady state, this condition would be fulfilled automatically. But in proving that a steady state exists, I must apply a fixed point argument to candidate distributions μ. The optimal policies, a_i, based on a distribution, μ, must tell the agent what to do given all possible histories. For the evolution of the economy given the policies $a = (a_i)_{i \in J \cup K}$ might be quite different from that predicted by μ, and give rise to histories not anticipated by μ. Hence, it is necessary to start with a μ that gives positive probability to all histories. Also, the new distribution $\mu(a)$ implied by the optimal actions must obey the same constraint. This means that rational behavior must make the economy vibrate through all possible paths, no matter what expectations may be. The conditions guaranteeing this vibration are complicated. No effort is made to make them natural and convincing.

The objective is to prove the following.

(19.1) Suppose that μ satisfies the complete expectations hypotheses (13.4)-(13.6) and the following conditions for $\eta > 0$ and $\gamma > 0$.

1) The quit rate expected by firms does not exceed γ.

2) Each consumer can expect that any firm will have a unit of output in the following period with probability at least η, no matter what the observational history of the consumer may be, provided that the current period is not a carnival period or the period just after a carnival period.

3) Each firm can expect that a customer will appear in the following period with probability at least η, provided that the current period is not a carnival period or the period just after a carnival period.

Let $a = (a_i)_{i \in J \cup K}$ be the optimal policy functions, given μ, and let $\mu(a)$ be the steady state distribution implied by a. Then there exist positive numbers α_{je}, $\alpha_j(H)$, and $\alpha(Z)$, for $j \in J$, $H \in \mathscr{H}_j^o$ and $Z \in \Omega$, which are independent of μ and such that $E_{\mu(a)}(c_{jen}) \geqq \alpha_{je}$, $\mu(a)([H_{jn}^o = H]) \geqq \alpha(H)$ and $\mu(a)([Z_n = Z]) \geqq \alpha(Z)$, for all $j \in J$, $H \in \mathscr{H}_j^o$ and $Z \in \Omega$. Furthermore, $\mu(a)$ satisfies conditions (1)-(3) above.

Conditions sufficient for (19.1) are given below. Many of these are quite complicated. The first of these is given in detail, as an example. I give only the idea of the others. $\varepsilon > 0$ is a fixed small number.

(19.2) For each $k \in K$, there exists for each $j \in J$ a set $E_j \subset (S_{0k} \setminus \{s_0^*, s_1^*, s_2^*\})$ with the following properties.

1) $\text{Prob}[s_{k,n+1} \in E_j | s_n] > (2\eta/|K|) + \varepsilon$ for all non-carnival states s_n.

2) If $s \in E_j$, then $\eta(U_{kb}(s,j) - 2p_j/|K|) - U_{kA}(s,(j,b)) > 0$.

3) $U_{kA}(s,c) > N$, for all $c \neq (j,b)$ or 0, where N exceeds the benefits that could be had from any other action.

Notice that the financial benefit from a job is bounded above by $\delta^{-1} w_j + w_{hj} + w_{fj}$, where δ is the probability of transition to state s_0^*. Hence, the N of (19.2) exists.

This condition would be summarized as follows. "For each $j \in J$, any consumer k for whom $\lambda_k \leqq 2/|K|$, will, during any non-carnival period, try to buy from firm j with probability at least $2\eta/|K|$.

(19.3) For each $j \in J$, any unemployed consumer k for whom $\lambda_k \geqq 1/2|K|$, will, during any non-carnival period, apply for a job at firm j with probability at least ε.

I assume that each firm has a productive state and a nonproductive state, among others. If it is in a productive state, it is profitable to hire if it has no workers, no matter what its inventories may be. If it is in a nonproductive state, it never wants to hire. (A productive state could not exist if there were not an upper bound (γ) on quit rates and a lower bound (η) on selling rates.)

(19.4) Any firm will during any non-carnival period enter a productive state with probability at least ε. Similarly, it will enter a non-productive state with probability at least ε.

Recall that the production function (18.6) is such that production is possible even if a firm has no employees. The following puts a lower bound on this production.

(19.5) For all $j \in J$, if $s_n \neq s_0^*$ or s_1^*, then
$\text{Prob}[G_{j2}(s_{j,n+1}) | s_n] > \eta$.

(19.6) Firms' spoilage functions L_j, (10.2), and production functions are such that any firm can in any period, with probability at least ε, lose all its stock of goods and have no production.

(19.7) The probability that any worker in any period would quit if he had a job is bounded above by γ.

(19.7) may be interpreted as a condition on $\text{Prob}[U_{ke}(s_{k,n+1}, j) \neq U_{ke}(s_{k,n}) | s_{kn}]$, for no worker would quit unless his attitude toward work changed.

(19.8) $|K| \geqq 2$ and $|J| \geqq 2$.

Notice that (19.8) implies that at least one consumer has $\lambda_k \geqq 1/2|K|$ and one other has $\lambda_k \leqq 2/|K|$.

I now prove (19.1). First consider H_{jn}. $H_{j,n+1}$ is made up of H_{jn} and (c^o_{jhn}, c^o_{jbn}). By (19.8), (19.2) and (19.3), each of the four possibilities for (c^o_{jhn}, c^o_{jbn}) occurs with probability at least ε^2, provided all

workers are unemployed. But by (19.4), all workers will be unemployed with probability at least ε^{JT}, where T is the time elapsed since the last carnival ended. (ε^{JT} is a lower bound on the probability that all firms have been in a nonproductive state since the last carnival.) It follows by induction on n that the probability of each of the possibilities of H_{jn}, conditional on the event that the last carnival ended T periods ago, is at least $\varepsilon^{(J+2)T}$. Since carnivals occur with probability δ in any period, each of the possibilities of H_{jn} occurs with probability at least $\delta\,\varepsilon^{(J+2)T}$. Hence, one may let $\alpha(H_{jn}) = \delta\,\varepsilon^{(J+2)T}$.

Similarly, consider Z_n. By (19.4)-(19.6), given Z_n any of the 4^J possibilities for z_n can occur with probability $\eta^J \varepsilon^{3J}$. Hence, by the argument just made $\alpha(Z_n)$ exists.

By (19.3) and (19.4), any unemployed worker k with $\lambda_k \geqq 1/2|K|$ will with probability at least ε^2 be hired in any non-carnival period. It follows easily that α_{je} as in (19.1) exists.

Finally, by (19.2), (19.5) and (19.7), the bounds η and γ apply to $\mu(a)$. This completes the proof of (19.1).

20. BOUNDARY CONDITIONS

Here, I state very strong conditions which guarantee that consumers' budget constraints can be satisfied. The first three assumptions say that all consumers are identical.

(20.1) $\qquad \theta_{jk} = |K|^{-1}$, for all k.

(20.2) The random variables $(s_{kn})_{k \in K}$ are independently and identically distributed and are independent of s_{jn} for all $j \in J$ and of s_{rn}.

(20.3) $U_{ke} = U_e$, $U_{kb} = U_b$, and $U_{kA} = U_A$, for all k.

21. THEOREMS

I assume that

(21.1) S_0 is a compact metric space and has the Borel structure induced by its topology. $\{s_0^*\}$, $\{s_1^*\}$, and $\{s_2^*\}$ are both closed and open in S_0.

(21.2) There exists at least one μ satisfying the conditions of (19.1).

(21.3) <u>Theorem</u>. Under these assumptions and those of sections 4 - 19, there exists a short-term equilibrium.

(21.4) <u>Theorem</u>. If one adjoins the assumptions of section 20, there exists a long-term equilibrium.

22. PROOF OF THEOREMS

I prove only theorem 21.4, since its proof contains the proof of (21.3).

Let $X_0 = \prod_{i \in J \cup K} (A_i \times C_i)$, so that $M = \prod_{n=-\infty}^{\infty} (S_0 \times X_0)$. Points in M will be denoted by (s, x), where $s \in S = \prod_{n=-\infty}^{\infty} S_0$ and $x \in \prod_{n=-\infty}^{\infty} X_0$. The n^{th} component of (s, x) is $(s, x)_n = (s_n, x_n)$.

Notice that X_0 is a finite set, so that by (21.1), $S_0 \times X_0$ is a compact metric space. Give $M = \prod_{n=-\infty}^{\infty} (S_0 \times X_0)$ a metric which induces the product topology. Then by the Tychonoff theorem, M is a compact metric space. Let W_1 be the set of all Borel probability measures in M. Give W_1 the weak topology. That is, a sequence μ_n in W_1 converges to μ if and only if $\int f\, d\mu_n$ converges to $\int f\, d\mu$ for all real-valued continuous functions f defined on M. Then, W_1 is a compact convex metric space [10], p. 45. Let W_2 be the set of all economic steady states. That is, W_2 is the set of all stationary measures on W_1 which are consistent with ν; or $W_2 = \{\mu \in W_1 : \mu$ satisfies (7.1) and (7.2)$\}$. It is easy to see that W_2 is convex and closed and hence compact.

I now show that the set of all steady states satisfying (19.1) is a compact convex set. First notice that if E is a subset of M defined by finitely many conditions on x_n, or of the form $\{s_n = s_i^*\}$, i = 0, 1, 2, then the indicator function of E is a continuous function M, so that $\mu(E)$ is continuous on W_2. This follows from the fact that X_0 has the discrete topology and the sets $\{s_i^*\}$ are topological components of S_0 (by 21.1). Next notice that the events $[H_{jn}^o = H]$ and $[Z_n = Z]$ are of the above form. Similarly, the functions c_{jqn}, c_{jen}, c_{jbn}^o, and c_{jsn} are random variables on M depending only on x_n. It follows immediately from the formula given below that the set, W, of steady states satisfying (19.1) is compact and convex.

$$W = \Big\{ \mu \in W_2 \,\Big|\, E_\mu(c_{jen}) \geqq \alpha_{je},\ \mu([H^o_{jn} = H]) \geqq \alpha(H) \text{ and}$$

$\mu([Z_n = Z]) \geqq \alpha(Z)$, for all $j \in J$, $H \in \mathscr{H}^o_j$ and $Z \in \Omega$;

$E_\mu(c_{jqn}) \geqq \gamma E_\mu(c_{jen})$, for all $j \in J$;

$\mu([Z_n = Z]$ and $[c_{jsn} > 0]) \geqq \eta\mu([Z_n = Z])$, for all $j \in J$ and for all $Z \in \Omega$ which indicate that the last carnival ended at least two periods ago; and

$\mu([H^o_{jn} = H]$ and $[c^o_{jbn} = 1]) \geqq \eta\mu([H^o_{jn} = H])$, for all $j \in J$ and for all $H \in \mathscr{H}^o_j$ which indicate that the last carnival ended at least two periods ago. $\Big\}$

Let $\Lambda = \{(\lambda_k)_{k \in K} = \lambda_k \geqq 0$, for all k and $\sum_{k \in K} \lambda_k = 1\}$. I make a fixed point argument on $W \times \Lambda$.

Given $\mu \in W$ and $\lambda \in \Lambda$, let $\sigma(\mu, \Lambda)$ be the unique steady state distribution generated by the almost surely unique policies $a(\mu,\lambda) = (a_i(\mu,\lambda))_{i \in J \cup K}$ determined by μ and λ. (See sections 14, 15 and 18.) By (19.1), $\sigma(\mu,\lambda) \in W$. It follows easily from the almost sure uniqueness of $a(\mu,\lambda)$ that $\sigma(\mu,\lambda)$ is well-defined.

I must show that $\sigma(\mu,\lambda)$ depends continuously on μ and λ. If μ_n converges to μ, then the expectations defined by μ_n converge to those defined by μ (see section 13 for the definition of expectations). For instance, the expected quit rate, $E_{\mu_n} c_{jqn}/E_{\mu_n} c_{jen}$, converges; the expected rate of sales, $E_\mu(c^o_{jbn} \,|\, H^o_{jn} = H)$ converges; etc. Here, I use the same argument I used to prove that W is closed. It follows easily from the definition of optimal policies that if λ_n converges to λ, then

(22.1) for every $i \in J \cup K$ and for every $H \in \mathscr{H}_i$, $a_i(\mu_n,\lambda_n)(s,H)$ converge almost everywhere to $a_i(\mu,\lambda)(s,H)$.

Here, I use the following facts. 1) Each agent's decision problem is really a Markov decision process. 2) The expectations define the transition probabilities of this process. 3) λ_k appears in the reward of consumer k's decision process and nowhere else.

By a routine argument, (22.1) implies that $\sigma(\mu_n,\lambda_n)$ converges to $\sigma(\mu,\lambda)$.

Now consider the net revenue functions $b_k(\mu)$ of the consumers (see 16.1). $b_k(\mu)$ is the expectation of variables depending only on x_n. Hence, $b_k(\mu)$ is a continuous function of μ. Notice that $\sum_{k\in K} b_k(\mu) = 0$.

Let $T: W \times \Lambda \to W \times \Lambda$ be defined by $T(\eta,\lambda) = (\sigma(\eta,\lambda),\lambda(\mu,\lambda))$, where $\lambda_k(\mu,\lambda)$ is defined as follows.

$$\lambda_k(\mu,\lambda) = \frac{\max(\lambda_k - b_k(\mu),0)}{\sum_{k\in K} \max(\lambda_k - b_k(\mu),0)} .$$

$\lambda_k(\mu,\lambda)$ is well-defined since $\sum_{k\in K} b_k(\mu) = 0$. T is clearly continuous. By (21.2), $W \times \Lambda$ is non-empty. Since $W \times \Lambda$ is a convex, compact subset of a locally convex, linear, topological space, I may apply the Tychonoff fixed point theorem [7], p. 456. Let $(\bar{\mu},\bar{\lambda})$ be a fixed point of T.

I now show that $b_k(\bar{\mu}) = 0$, for all k. If this were not so, then $b_{k_1}(\bar{\mu}) > 0$, for some k_1. By the nature of the map T, $\lambda_{k_1} = 0$. There must be some consumer k_2 with $\lambda_{k_2} > 0$. By (20.2) and (20.3), consumer k_2 would earn at least as much wage income and spend at most as much, on the average, as would consumer k_1. By (20.1), the dividend incomes

of k_1 and k_2 are the same. Therefore, $b_{k_2}(\bar{\mu}) \geqq b_{k_1}(\bar{\mu}) > 0$. By the nature of the map T, $\lambda_{k_2} = 0$. This contradiction proves that $b_k(\bar{\mu}) = 0$ for all k.

It should be clear that $(\bar{\mu}, \bar{\lambda})$ is a long-term equilibrium.

Q.E.D.

23. THE INDETERMINACY OF EQUILIBRIUM

As has been emphasized in the introduction, rational expectations equilibria are not unique. It might seem that the lack of uniqueness arises only because of the lack of forward markets in an intertemporal model. However, the example of the next section demonstrates that there may be many such equilibria even in a situation in which forward markets are not called for. In the example, no storage takes place and future supplies and demands are statistically independent of current conditions. Nevertheless, there are two stable equilibria, one of which dominates the other according to the Pareto welfare criterion.

This indeterminacy reappears in a disturbing form when one considers the marginal utilities of money, λ_k. These are determined for each individual by his long-run budget constraint. However, a normalization has been made, that the sum of the marginal utilities equals one. This sum can be thought of as a collective measure of the value of money. Nothing in the model determines this. Increasing the sum to, say, $1 + \varepsilon$, corresponds to dividing all prices by $1 + \varepsilon$. One could perfectly well have an equilibrium at these new prices, as the example below demonstrates. And the general welfare could be higher or lower at the new equilibrium. It is hard to think of any instrument that could affect the

general psychological expectation as to the value of money.

24. AN EXAMPLE

There are two firms, firms 1 and 2, and two consumers, consumers 1 and 2. Consumer 1 works only for firm 1 and buys only from firm 2. Consumer 2 works only for firm 2 and buys only from firm 1. Neither consumer, i, suffers a loss of utility if he asks firm i for a job, and there are no fees for getting a job or for quitting or firing. Output of firms lasts only one period and then perishes if it is not sold. The productivity of firms from period to period is independently and identically distributed. The same statement applies to the utilities consumers derive from consumption and the disutilities they suffer from working or shopping. Also, all these utilities, disutilities and productivities are statistically independent of each other. Suppose that each consumer receives half the profit flow of each firm.

Let U_{ien} be the disutility consumer i suffers from working in period n and let the probability density of U_{ien} be

$$\rho_e(t) = \begin{cases} 1/3\ , & \text{if } 0 \leq t \leq 3 \\ 0\ , & \text{otherwise}\ . \end{cases}$$

Consumer i offers to work in period n if $U_{ien} \leqq \lambda w$, where λ is his marginal utility of money and w is the wage. Let $\lambda = 1$ and $w = 2$ and let α_w be the probability a consumer offers to work in any period. Then, $\alpha_w = 2/3$.

Let x_{in} be the expectation of firm i in period n of the probability that it will produce one unit of output in the following period if it hires one

worker in the current period. Suppose that the x_{in} are independently and identically distributed according to the density function

$$\rho_p(t) = \begin{cases} t^{-3}/4\,, & \text{if } 1/3 \leq t \leq 1\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

Let α_s be the probability that a firm makes a sale in any period and let α_b be the probability that a firm has a unit of output for sale (or that a consumer can buy) in any period. Then, a firm will offer to hire a worker only if $\alpha_s x_{ni} p > w$ or $x_{ni} > (\alpha_s p/w)^{-1}$, where p is the price of a firm's output. Let p = 6. If a firm tries to hire, it receives a worker with probability α_w. Hence, the probability that a firm produces in any period is

$$\alpha_b = \alpha_w \int_{(\alpha_s p/w)^{-1}}^{1} t\rho_p(t)\,dt = \frac{2}{3}\int_{(3\alpha_s)^{-1}}^{1} \frac{t^{-2}}{4}\,dt\,.$$

Hence,

$$\alpha_b(\alpha_s) = \begin{cases} \frac{1}{2}(\alpha_s - \frac{1}{3})\,, & \text{if } \frac{1}{3} \leqq \alpha_s \leq 1\,, \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{24.1}$$

Let the disutility that a consumer suffers from shopping be one and let U_{icn} be the utility that consumer i derives from consuming one unit of good in period n. Consumer i goes shopping in period n if $U_{icn} > \alpha_b^{-1} + \lambda_p - \alpha_b^{-1} + 6$. Let U_{icn} be distributed according to a density function ρ_c satisfying

$$\rho_c(t) = [2 + \frac{9}{5}\sin(18\pi(t-6))](t-6)^{-2}\,,$$

$$\text{if } 9 + \frac{3}{11} \leqq t \leqq 42, \text{ and } \int_0^\infty \rho_c(t)\,dt = 1.$$

Then, the probability that a consumer goes shopping is

$$\alpha_s = \int_{\alpha_b^{-1}+6}^{\infty} \rho_c(t)\,dt \text{ , and}$$

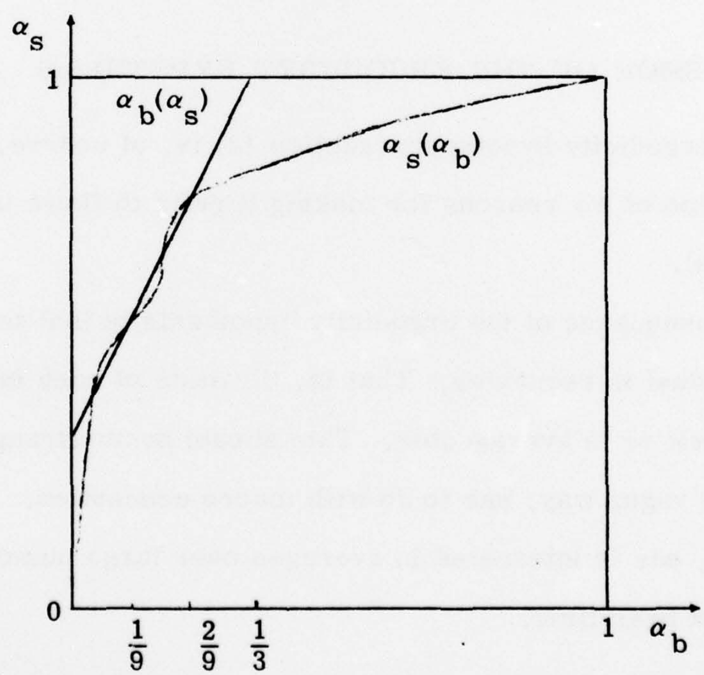
(24.2) $$\alpha_s(\alpha_b) = \frac{1}{3} + 2\alpha_b + \frac{1}{10\pi}\sin(18\pi\alpha_b)\text{ ,}$$

$$\text{if } \frac{1}{36} \leqq \alpha_b \leqq \frac{11}{36}.$$

Solving (24.1) for α_s in terms of α_b, I also obtain

(24.3) $$\alpha_s = \frac{1}{3} + 2\alpha_b\text{ , if } 0 \leqq \alpha_b \leqq \frac{1}{3}.$$

In a quantity equilibrium, both (24.2) and (24.3) must be satisfied, so that $\sin(18\pi\alpha_b) = 0$, which has solutions at $\alpha_b = \frac{n}{18}$, for n = 1, 2, 3, 4 and 5. Only the solutions $\alpha_b = \frac{1}{9}$ and $\frac{2}{9}$ can be considered to be stable. The stability may be read from the graph given below of the functions of (24.1) and (24.2).



The solutions corresponding to $\alpha_b = \frac{1}{9}$ and $\frac{2}{9}$ correspond to long-term equilibria, for by the symmetry of the model and of the solutions, the budget constraints are automatically satisfied. All income is spent and both consumers earn and spend the same amount.

The functions $\alpha_b(\alpha_s)$ and $\alpha_s(\alpha_b)$ are largely arbitrary. In fact, $\alpha_b(\alpha_s)$ is completely arbitrary, except that it must carry zero to zero and never have a negative shape. Hence, it is possible to have multiple equilibria or none at all.

The equilibrium $\alpha_b = \frac{2}{9}$ is clearly superior in terms of average utility flow to the equilibrium $\alpha_b = \frac{1}{9}$.

Increasing λ, the common marginal utility of money of the consumers, moves the function $\alpha_b(\alpha_s)$ to the right, in the diagram. It also moves the function $\alpha_s(\alpha_b)$ downward. In general, the resulting effect on average utility flow is ambiguous, as is the effect on the values of α_b and α_s.

25. DISCUSSION OF THE ERGODICITY HYPOTHESIS

The ergodicity hypothesis (section 12) is, of course, absurd. Consideration of my reasons for making it point to flaws in the conception of the model.

A consequence of the ergodicity hypothesis is that the behavior of each individual is recursive. That is, the state of each individual comes back on itself or is averageable. This should seem strange in a model which, in a vague way, has to do with macro economics. For in macro economics, one is interested in averages over large numbers of individuals, not over time.

The ergodicity was needed mainly for two reasons. First of all, the budget constraint requires a definition of the average net revenue of each consumer. The consumer is interested in his long-run average revenue. But mathematically one must deal with his expected revenue in one period. These two measures of net revenue flow are the same if the steady state distribution is ergodic (see section 16). Otherwise, they might not be.

A second reason for the ergodicity hypothesis is that it avoids technical problems brought on by rational expectations. Imagine that any time a general shortage of good A is observed, all agents expect a high demand and supply of good B, N periods later. This expectation could be self-fulfilling. A "market day" for B would have been signaled. Hence, such behavior must be admissible in the candidate steady state distributions, μ. But there need be no bound on N. In fact, a candidate μ might incorporate infinitely many such forms of expectations, with unbounded N's. Such a μ might give rise to optimal action functions, $a = (a_i)_{i \in J \cup K}$, which would incorporate these unbounded lags. But then, there might not be a unique steady state distribution $\mu(a)$ generated by a. The economy might be induced to anticipate ever further into the future by a suitable choice of initial conditions. Of course, this possibility might be avoided by assuming that μ is itself ergodic. But ergodicity is not a property which is closed in the weak topology. So one is forced to control the lag in expectations. This is done by the ergodicity hypothesis, for all may be forgotten after a carnival.

All these problems could be avoided if one dropped rational expectations and the budget constraint. If one interprets the model as a snap shot of the economy over a short interval of time, then there is no reason that

budgets should balance and it still makes sense to assume that the marginal utilities of money are nearly constant. Expectations must be "reasonable," of course.

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13. ABSTRACT

This paper demonstrates the existence of a rational expectations, quantity equilibrium in a general equilibrium model of an economy with market fritions. A market is said to have frictions if buyers and sellers have trouble finding each other, if it is costly for them to search for each other, and if it is costly to wait to buy or sell. Equilibrium is a stationary probability distribution over the set of possible time paths of states of the economy. This equilibrium reflects rational expectations if all agents know the stationary distribution of the variables they observe and if they exploit this information. Prices are fixed and are not necessarily equilibrium prices in any sense.

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